Dark Energy: Mystery of the Millennium

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Abstract.
Nearly seventy per cent of the energy density in the universe is unclustered and exerts negative pressure. This conclusion — now supported by numerous observations — poses the greatest challenge for theoretical physics today. I discuss this issue with special emphasis on the cosmological constant as the possible choice for the dark energy. Several curious features of a universe with a cosmological constant are described and some possible approaches to understand the nature of the cosmological constant are reviewed. In particular, I show how some of the recent ideas, related to a thermodynamic route to gravity, allow us to: (i) create a paradigm in which the bulk value of cosmological constant is irrelevant and (ii) obtain the correct, observed, value for the cosmological constant from vacuum fluctuations in a region confined by the deSitter horizon.

Keywords: cosmological constant, dark energy, Einstein-Hilbert action, Gauss-Bonnet, Holography

1. THE RISE OF THE DARK ENERGY: BRIEF HISTORY

The cosmological data of exquisite quality, which became available in the last couple of decades, have thrusted upon us a rather preposterous composition for the universe which defies any simple explanation, thereby posing the greatest challenge theoretical physics has ever faced. It is conventional to measure the energy densities of the various species which drive the expansion of the universe in terms of a *critical energy density* $\rho_c = 3H_0^2/8\pi G$ where $H_0 = (\dot{a}/a)_0$ is the rate of expansion of the universe at present. The variables $\Omega_i = \rho_i/\rho_c$ will then give the fractional contribution of different components of the universe (i denoting baryons, dark matter, radiation, etc.) to the critical density. Observations suggest that the universe has $0.98 \lesssim \Omega_{tot} \lesssim 1.08$ with radiation (R), baryons (B), dark matter, made of weakly interacting massive particles (DM) and dark energy (DE) contributing $\Omega_R \simeq 5\times 10^{-5}, \Omega_B \simeq 0.04, \Omega_{DM} \simeq 0.26, \Omega_{DE} \simeq 0.7$, respectively. All known observations [1, 2, 3] are consistent with such an — admittedly weird — composition for the universe.

Among all these components, the dark energy, which exerts negative pressure, is probably the weirdest. And nobody really wanted it! To understand its rapid acceptance by the community one needs to look at its recent history briefly. Early analysis of several observations [4] indicated that this component is unclustered and has negative pressure — the observation which made me personally sit up and take note being the APM result. This is confirmed dramatically by the supernova observations[5]. The current observations suggest that this component has $w = p/\rho \lesssim -0.78$ and contributes $\Omega_{DE} \cong 0.60 - 0.75$; for a critical look at the current data, see [6].

While the composition of the universe is puzzling — and I will concentrate on the *unknown*, puzzling aspects of our universe for the rest of the talk — it should not prevent us from appreciating the remarkable *successes* of the standard cosmological paradigm. The key idea is that if there existed small fluctuations in the energy density in the early universe, then gravitational instability can amplify them in a well-understood manner leading to structures like galaxies etc. today. The most popular model for generating these fluctuations is based on the idea that if the very early universe went through an inflationary phase [7], then the quantum fluctuations of the field driving the inflation can lead to energy density fluctuations[8, 9]. It is possible to construct models of inflation such that these fluctuations are described by a Gaussian random field and are characterized by a power spectrum of the form $P(k) = Ak^n$ with $n \simeq 1$. The models cannot predict the value of the amplitude A in an unambiguous manner but it can be determined from CMBR observations. The CMBR observations are consistent with the inflationary model for the generation of perturbations and gives $A \simeq (28.3h^{-1}Mpc)^4$ and $n = 0.97 \pm 0.023$. (The first results were from COBE [10] and WMAP has re-confirmed them with far greater accuracy). When the perturbation is small, one can use well defined linear perturbation theory to study its growth [11]. But when $\delta \approx (\delta\rho/\rho)$ is comparable to unity the perturbation theory breaks down. Since there is more power at small scales, smaller scales go non-linear first and structure forms hierarchically. The non linear evolution of the *dark matter halos* (which is an example of statistical mechanics of

self gravitating systems; see e.g.[12]) can be understood by simulations as well as theoretical models based on approximate ansatz [13] and nonlinear scaling relations [14]. The baryons in the halo will cool and undergo collapse in a fairly complex manner because of gas dynamical processes. It seems unlikely that the baryonic collapse and galaxy formation can be understood by analytic approximations; one needs to do high resolution computer simulations to make any progress [15]. The results obtained from all these attempts are broadly consistent with observations. So, to the zeroth order, the universe is characterized by just seven numbers: $h \approx 0.7$ describing the current rate of expansion; $\Omega_{DE} \simeq 0.7, \Omega_{DM} \simeq 0.26, \Omega_B \simeq 0.04, \Omega_R \simeq 5 \times 10^{-5}$ giving the composition of the universe; the amplitude $A \simeq (28.3h^{-1}Mpc)^4$ and the index $n \simeq 1$ of the initial perturbations. Establishing this cosmological paradigm is a remarkable progress by any sensible criterion.

The remaining challenge, of course, is to make some sense out of these numbers from a more fundamental point of view. It is rather frustrating that the only component of the universe which we understand theoretically is the radiation! While understanding the baryonic and dark matter components [in particular the values of Ω_B and Ω_{DM}] is by no means trivial, the issue of dark energy is lot more perplexing, thereby justifying the attention it has received recently.

The key observational feature of dark energy is that — treated as a fluid with a stress tensor $T^a_b =$ dia $(\rho, -p, -p, -p)$ — it has an equation state $p = w\rho$ with $w \lesssim -0.8$ at the present epoch. The spatial part $\mathbf{g}$ of the geodesic acceleration (which measures the relative acceleration of two geodesics in the spacetime) satisfies an *exact* equation in general relativity given by:

$$\nabla \cdot \mathbf{g} = -4\pi G(\rho + 3p) \tag{1}$$

This shows that the source of geodesic acceleration is $(\rho + 3p)$ and not ρ. As long as $(\rho + 3p) > 0$, gravity remains attractive while $(\rho + 3p) < 0$ can lead to repulsive gravitational effects. In other words, dark energy with sufficiently negative pressure will accelerate the expansion of the universe, once it starts dominating over the normal matter. This is precisely what is established from the study of high redshift supernova, which can be used to determine the expansion rate of the universe in the past [5].

The simplest model for a fluid with negative pressure is the cosmological constant (for a sample of recent reviews, see [16]) with $w = -1, \rho = -p =$ constant. If the dark energy is indeed a cosmological constant, then it introduces a fundamental length scale in the theory $L_\Lambda \equiv H_\Lambda^{-1}$, related to the constant dark energy density $\rho_{\rm DE}$ by $H_\Lambda^2 \equiv (8\pi G\rho_{\rm DE}/3)$. In classical general relativity, based on the constants G, c and L_Λ, it is not possible to construct any dimensionless combination from these constants. But when one introduces the Planck constant, $\hbar$, it is possible to form the dimensionless combination $H_\Lambda^2(G\hbar/c^3) \equiv (L_P^2/L_\Lambda^2)$. Observations then require $(L_P^2/L_\Lambda^2) \lesssim 10^{-123}$. As has been mentioned several times in literature, this will require enormous fine tuning. What is more, in the past, the energy density of normal matter and radiation would have been higher while the energy density contributed by the cosmological constant does not change. Hence we need to adjust the energy densities of normal matter and cosmological constant in the early epoch very carefully so that $\rho_\Lambda \gtrsim \rho_{\rm NR}$ around the current epoch. This raises the second of the two cosmological constant problems: Why is $(\rho_\Lambda/\rho_{\rm NR}) = \mathscr{O}(1)$ at the *current* phase of the universe ?

2. SCALAR FIELDS: THE ‘DENIAL’ APPROACH TO COSMOLOGICAL CONSTANT

Because of these conceptual problems associated with the cosmological constant, people have explored a large variety of alternative possibilities. The most popular among them uses a scalar field ϕ with a suitably chosen potential $V(\phi)$ so as to make the vacuum energy vary with time. The hope then is that, one can find a model in which the current value can be explained naturally without any fine tuning. A simple form of the source with variable w are scalar fields with Lagrangians of different forms, of which we will discuss two possibilities:

$$L_{\rm quin} = \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi); \quad L_{\rm tach} = -V(\phi)[1 - \partial_a\phi\partial^a\phi]^{1/2} \tag{2}$$

Both these Lagrangians involve one arbitrary function $V(\phi)$. The first one, $L_{\rm quin}$, which is a natural generalization of the Lagrangian for a non-relativistic particle, $L = (1/2)\dot{q}^2 - V(q)$, is usually called quintessence (for a small sample of models, see [17]). When it acts as a source in Friedman universe, it is characterized by a time dependent $w(t)$ with

$$\rho_q(t) = \frac{1}{2}\dot{\phi}^2 + V; \quad p_q(t) = \frac{1}{2}\dot{\phi}^2 - V; \quad w_q = \frac{1 - (2V/\dot{\phi}^2)}{1 + (2V/\dot{\phi}^2)} \tag{3}$$

The structure of the second Lagrangian in Eq. (2) (which arises in string theory [18]) can be understood by a simple analogy from special relativity. A relativistic particle with (one dimensional) position $q(t)$ and mass m is described by the Lagrangian $L = -m\sqrt{1-\dot{q}^2}$. It has the energy $E = m/\sqrt{1-\dot{q}^2}$ and momentum $k = m\dot{q}/\sqrt{1-\dot{q}^2}$ which are related by $E^2 = k^2 + m^2$. As is well known, this allows the possibility of having *massless* particles with finite energy for which $E^2 = k^2$. This is achieved by taking the limit of $m \to 0$ and $\dot{q} \to 1$, while keeping the ratio in $E = m/\sqrt{1-\dot{q}^2}$ finite. The momentum acquires a life of its own, unconnected with the velocity $\dot{q}$, and the energy is expressed in terms of the momentum (rather than in terms of $\dot{q}$) in the Hamiltonian formulation. We can now construct a field theory by upgrading $q(t)$ to a field ϕ. Relativistic invariance now requires ϕ to depend on both space and time [$\phi = \phi(t, \mathbf{x})$] and $\dot{q}^2$ to be replaced by $\partial_i\phi\partial^i\phi$. It is also possible now to treat the mass parameter m as a function of ϕ, say, $V(\phi)$ thereby obtaining a field theoretic Lagrangian $L = -V(\phi)\sqrt{1-\partial^i\phi\partial_i\phi}$. The Hamiltonian structure of this theory is algebraically very similar to the special relativistic example we started with. In particular, the theory allows solutions in which $V \to 0$, $\partial_i\phi\partial^i\phi \to 1$ simultaneously, keeping the energy (density) finite. Such solutions will have finite momentum density (analogous to a massless particle with finite momentum k) and energy density. Since the solutions can now depend on both space and time (unlike the special relativistic example in which q depended only on time), the momentum density can be an arbitrary function of the spatial coordinate. The structure of this Lagrangian is similar to those analyzed in a wide class of models called *K-essence* [19] and provides a rich gamut of possibilities in the context of cosmology [20, 21].

Since the quintessence field (or the tachyonic field) has an undetermined free function $V(\phi)$, it is possible to choose this function in order to produce a given $H(a)$. To see this explicitly, let us assume that the universe has two forms of energy density with $\rho(a) = \rho_{\rm known}(a) + \rho_\phi(a)$ where $\rho_{\rm known}(a)$ arises from any known forms of source (matter, radiation, ...) and $\rho_\phi(a)$ is due to a scalar field. Let us first consider quintessence. Here, the potential is given implicitly by the form [22, 20]

$$V(a) = \frac{1}{16\pi G} H(1-Q)\left[6H + 2aH' - \frac{aHQ'}{1-Q}\right] \tag{4}$$

$$\phi(a) = \left[\frac{1}{8\pi G}\right]^{1/2} \int \frac{da}{a}\left[aQ' - (1-Q)\frac{d\ln H^2}{d\ln a}\right]^{1/2} \tag{5}$$

where $Q(a) \equiv [8\pi G\rho_{\rm known}(a)/3H^2(a)]$ and prime denotes differentiation with respect to a. Given any $H(a), Q(a)$, these equations determine $V(a)$ and $\phi(a)$ and thus the potential $V(\phi)$. *Every quintessence model studied in the literature can be obtained from these equations.*

Similar results exists for the tachyonic scalar field as well [20]. For example, given any $H(a)$, one can construct a tachyonic potential $V(\phi)$ so that the scalar field is the source for the cosmology. The equations determining $V(\phi)$ are now given by:

$$\phi(a) = \int \frac{da}{aH}\left(\frac{aQ'}{3(1-Q)} - \frac{2}{3}\frac{aH'}{H}\right)^{1/2} \tag{6}$$

$$V(a) = \frac{3H^2}{8\pi G}(1-Q)\left(1 + \frac{2}{3}\frac{aH'}{H} - \frac{aQ'}{3(1-Q)}\right)^{1/2} \tag{7}$$

Equations (6) and (7) completely solve the problem. Given any $H(a)$, these equations determine $V(a)$ and $\phi(a)$ and thus the potential $V(\phi)$. A wide variety of phenomenological models with time dependent cosmological constant have been considered in the literature; all of these can be mapped to a scalar field model with a suitable $V(\phi)$.

While the scalar field models enjoy considerable popularity (one reason being they are easy to construct!) it is very doubtful whether they have helped us to understand the nature of the dark energy at any deeper level. These models, viewed objectively, suffer from several shortcomings:

- They completely lack predictive power. As explicitly demonstrated above, virtually every form of $a(t)$ can be modeled by a suitable "designer" $V(\phi)$.
- These models are degenerate in another sense. The previous discussion illustrates that even when $w(a)$ is known/specified, it is not possible to proceed further and determine the nature of the scalar field Lagrangian. The explicit examples given above show that there are *at least* two different forms of scalar field Lagrangians (corresponding to the quintessence or the tachyonic field) which could lead to the same $w(a)$. (See the first paper in ref.[6] for an explicit example of such a construction.)

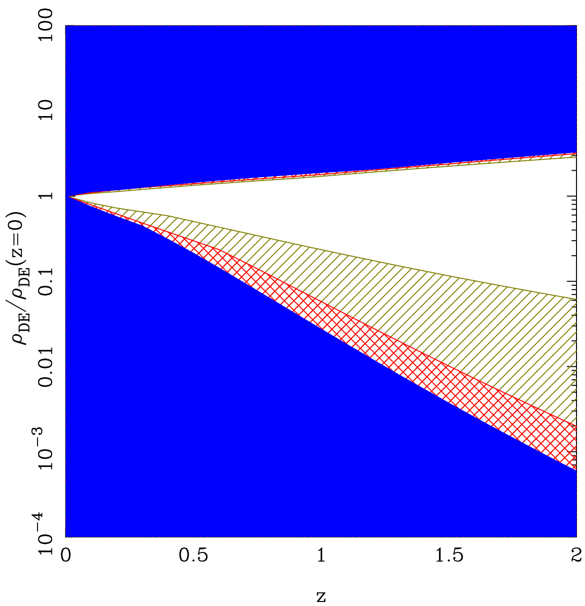

FIGURE 1. The observational constraints on the variation of dark energy density as a function of redshift from WMAP and SNLS data (see [23]). The green/hatched region is excluded at 68% confidence limit, red/cross-hatched region at 95% confidence level and the blue/solid region at 99% confidence limit. The white region shows the allowed range of variation of dark energy at 68% confidence limit.

- All the scalar field potentials require fine tuning of the parameters in order to be viable. This is obvious in the quintessence models in which adding a constant to the potential is the same as invoking a cosmological constant. So to make the quintessence models work, *we first need to assume the cosmological constant is zero.* These models, therefore, merely push the cosmological constant problem to another level, making it somebody else's problem!.
- By and large, the potentials used in the literature have no natural field theoretical justification. All of them are non-renormalisable in the conventional sense and have to be interpreted as a low energy effective potential in an ad hoc manner.
- One key difference between cosmological constant and scalar field models is that the latter lead to a $w(a)$ which varies with time. If observations have demanded this, or even if observations have ruled out $w = -1$ at the present epoch, then one would have been forced to take alternative models seriously. However, all available observations are consistent with cosmological constant ($w = -1$) and — in fact — the possible variation of w is strongly constrained [23] as shown in Figure 1.
- While on the topic of observational constraints on $w(t)$, it must be stressed that: (a) There is fair amount of tension between WMAP and SN-Gold data and one should be very careful about the priors used in these analysis. The recent SNLS data [24] is more concordant with WMAP than the SN Gold data. (b) There is no observational evidence for $w < -1$. (For more details related to these issues, see the last reference in [23].)

Given this situation, we shall now take a more serious look at the cosmological constant as the source of dark energy in the universe.

3. COSMOLOGICAL CONSTANT: FACING UP TO THE CHALLENGE

The observational and theoretical features described above suggests that one should consider cosmological constant as the most natural candidate for dark energy. Though it leads to well know fine tuning problems, it also has certain attractive features that need to kept in mind.

- Cosmological constant is the most economical [just one number] and simplest explanation for all the observations. I repeat that there is absolutely *no* evidence for variation of dark energy density with redshift, which is consistent

with the assumption of cosmological constant .

- Once we invoke the cosmological constant, classical gravity will be described by the three constants G, c and $\Lambda \equiv L_\Lambda^{-2}$. It is not possible to obtain a dimensionless quantity from these; so, within classical theory, there is no fine tuning issue. Since $\Lambda(G\hbar/c^3) \equiv (L_P/L_\Lambda)^2 \approx 10^{-123}$, it is obvious that the cosmological constant is telling us something regarding *quantum gravity*, indicated by the combination $G\hbar$. *An acid test for any quantum gravity model will be its ability to explain this value;* needless to say, all the currently available models — strings, loops etc. — flunk this test. Even several different approaches to semiclassical gravity [25] are silent about cosmological constant.
- If dark energy is indeed cosmological constant this will be the greatest contribution from cosmology to fundamental physics. It will be unfortunate if we miss this chance by invoking some scalar field epicycles!

In this context, it is worth stressing another peculiar feature of cosmological constant when it is treated as a clue to quantum gravity. It is well known that, based on energy scales, the cosmological constant problem is an infra red problem *par excellence*. At the same time, it is a relic of a quantum gravitational effect (or principle) of unknown nature. An analogy will be helpful to illustrate this point [26]. Suppose you solve the Schrodinger equation for the Helium atom for the quantum states of the two electrons $\psi(x_1, x_2)$. When the result is compared with observations, you will find that only half the states — those in which $\psi(x_1, x_2)$ is antisymmetric under $x_1 \longleftrightarrow x_2$ interchange — are realized in nature. But the low energy Hamiltonian for electrons in the Helium atom has no information about this effect! Here is a low energy (IR) effect which is a relic of relativistic quantum field theory (spin-statistics theorem) that is totally non perturbative, in the sense that writing corrections to the Helium atom Hamiltonian in some $(1/c)$ expansion will *not* reproduce this result. I suspect the current value of cosmological constant is related to quantum gravity in a similar way. There must exist a deep principle in quantum gravity which leaves its non perturbative trace even in the low energy limit that appears as the cosmological constant (see Sections 5, 6).

Let us now turn our attention to few of the many attempts to understand the cosmological constant with the choice dictated by personal bias. A host of other approaches exist in literature, some of which can be found in [27].

3.1. Geometrical Duality in our Universe

Before we discuss the ideas to explain the cosmological constant, it is important to realise some peculiar features which arise in a universe which has two independent length scales. A universe with two length scales L_Λ and L_P will be asymptotically De Sitter with $a(t) \propto \exp(t/L_\Lambda)$ at late times. Given the two length scales L_P and L_Λ, one can construct two energy scales $\rho_{\rm UV} = 1/L_P^4$ and $\rho_{\rm IR} = 1/L_\Lambda^4$ in natural units ($c = \hbar = 1$). There is sufficient amount of justification from different theoretical perspectives to treat L_P as the zero point length of spacetime [28], giving a natural interpretation to $\rho_{\rm UV}$. The second one, $\rho_{\rm IR}$ also has a natural interpretation. The universe which is asymptotically De Sitter has a horizon and associated thermodynamics [29] with a temperature $T = H_\Lambda/2\pi$ and the corresponding thermal energy density $\rho_{thermal} \propto T^4 \propto 1/L_\Lambda^4 = \rho_{\rm IR}$. Thus L_P determines the *highest* possible energy density in the universe while L_Λ determines the *lowest* possible energy density in this universe. As the energy density of normal matter drops below this value, the thermal ambience of the De Sitter phase will remain constant and provide the irreducible ‘vacuum noise’. *Note that the dark energy density is the the geometric mean $\rho_{\rm DE} = \sqrt{\rho_{\rm IR}\rho_{\rm UV}}$ between the two energy densities.* If we define a dark energy length scale L_{DE} such that $\rho_{\rm DE} = 1/L_{DE}^4$ then $L_{DE} = \sqrt{L_P L_\Lambda}$ is the geometric mean of the two length scales in the universe. (Incidentally, $L_{DE} \approx 0.04$ mm is macroscopic; it is also pretty close to the length scale associated with a neutrino mass of 10^{-2} eV; another intriguing coincidence ?!)

Figure 2 describes some peculiar features in such a universe [30, 31]. Using the characteristic length scale of expansion, the Hubble radius $d_H \equiv (\dot{a}/a)^{-1}$, we can distinguish between three different phases of such a universe. The first phase is when the universe went through a inflationary expansion with $d_H =$ constant; the second phase is the radiation/matter dominated phase in which most of the standard cosmology operates and d_H increases monotonically; the third phase is that of re-inflation (or accelerated expansion) governed by the cosmological constant in which d_H is again a constant. The first and last phases are time translation invariant; that is, $t \rightarrow t+$ constant is an (approximate) invariance for the universe in these two phases. The universe satisfies the perfect cosmological principle and is in steady state during these phases!

In fact, one can easily imagine a scenario in which the two deSitter phases (first and last) are of arbitrarily long duration [30]. If $\Omega_\Lambda \approx 0.7, \Omega_{DM} \approx 0.3$ the final deSitter phase *does* last forever; as regards the inflationary phase, nothing prevents it from lasting for arbitrarily long duration. Viewed from this perspective, the in between phase —

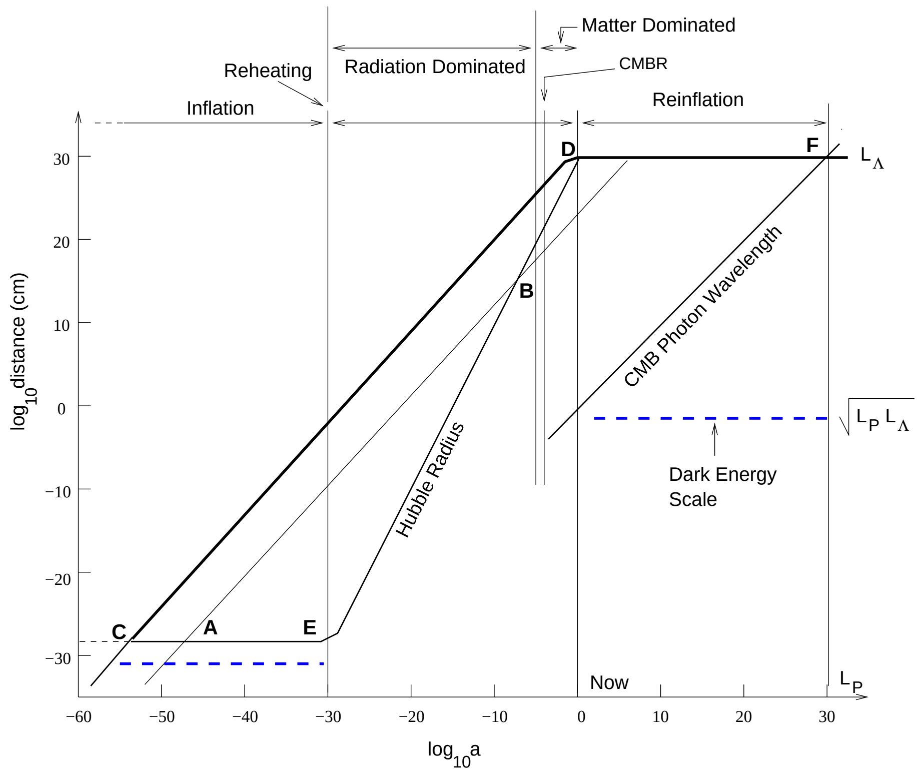

FIGURE 2. The geometrical structure of a universe with two length scales L_P and L_Λ corresponding to the Planck length and the cosmological constant [30, 31]. Such a universe spends most of its time in two De Sitter phases which are (approximately) time translation invariant. The first De Sitter phase corresponds to the inflation and the second corresponds to the accelerated expansion arising from the cosmological constant. Most of the perturbations generated during the inflation will leave the Hubble radius (at some A, say) and re-enter (at B). However, perturbations which exit the Hubble radius earlier than C will never re-enter the Hubble radius, thereby introducing a specific dynamic range CE during the inflationary phase. The epoch F is characterized by the redshifted CMB temperature becoming equal to the De Sitter temperature $(H_\Lambda/2\pi)$ which introduces another dynamic range DF in the accelerated expansion after which the universe is dominated by vacuum noise of the De Sitter spacetime.

in which most of the ‘interesting’ cosmological phenomena occur — is of negligible measure in the span of time. It merely connects two steady state phases of the universe. The figure 2 also shows the variation of L_{DE} by broken horizontal lines.

While the two deSitter phases can last forever in principle, there is a natural cut off length scale in both of them which makes the region of physical relevance to be finite [30]. Let us first discuss the case of re-inflation in the late universe. As the universe grows exponentially in the phase 3, the wavelength of CMBR photons are being redshifted rapidly. When the temperature of the CMBR radiation drops below the deSitter temperature (which happens when the wavelength of the typical CMBR photon is stretched to the L_Λ.) the universe will be essentially dominated by the vacuum thermal noise of the deSitter phase. This happens at the point marked F when the expansion factor is $a = a_F$ determined by the equation $T_0(a_0/a_F) = (1/2\pi L_\Lambda)$. Let $a = a_\Lambda$ be the epoch at which cosmological constant started dominating over matter, so that $(a_\Lambda/a_0)^3 = (\Omega_{DM}/\Omega_\Lambda)$. Then we find that the dynamic range of DF is

$$\frac{a_F}{a_\Lambda} = 2\pi T_0 L_\Lambda \left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{1/3} \approx 3\times 10^{30} \tag{8}$$

Interestingly enough, one can also impose a similar bound on the physically relevant duration of inflation. We know that the quantum fluctuations generated during this inflationary phase could act as seeds of structure formation in

the universe [8]. Consider a perturbation at some given wavelength scale which is stretched with the expansion of the universe as $\lambda \propto a(t)$. (See the line marked AB in Figure 2.) During the inflationary phase, the Hubble radius remains constant while the wavelength increases, so that the perturbation will 'exit' the Hubble radius at some time (the point A in Figure 2). In the radiation dominated phase, the Hubble radius $d_H \propto t \propto a^2$ grows faster than the wavelength $\lambda \propto a(t)$. Hence, normally, the perturbation will 're-enter' the Hubble radius at some time (the point B in Figure 2). If there was no re-inflation, this will make *all* wavelengths re-enter the Hubble radius sooner or later. But if the universe undergoes re-inflation, then the Hubble radius 'flattens out' at late times and some of the perturbations will *never* reenter the Hubble radius! The limiting perturbation which just 'grazes' the Hubble radius as the universe enters the re-inflationary phase is shown by the line marked CD in Figure 2. If we use the criterion that we need the perturbation to reenter the Hubble radius, we get a natural bound on the duration of inflation which is of direct astrophysical relevance. This portion of the inflationary regime is marked by CE and can be calculated as follows: Consider a perturbation which leaves the Hubble radius (H_{in}^{-1}) during the inflationary epoch at $a = a_i$. It will grow to the size $H_{in}^{-1}(a/a_i)$ at a later epoch. We want to determine a_i such that this length scale grows to L_Λ just when the dark energy starts dominating over matter; that is at the epoch $a = a_\Lambda = a_0(\Omega_{DM}/\Omega_\Lambda)^{1/3}$. This gives $H_{in}^{-1}(a_\Lambda/a_i) = L_\Lambda$ so that $a_i = (H_{in}^{-1}/L_\Lambda)(\Omega_{DM}/\Omega_\Lambda)^{1/3}a_0$. On the other hand, the inflation ends at $a = a_{end}$ where $a_{end}/a_0 = T_0/T_{\rm reheat}$ where $T_{\rm reheat}$ is the temperature to which the universe has been reheated at the end of inflation. Using these two results we can determine the dynamic range of CE to be

$$\frac{a_{\rm end}}{a_i} = \left(\frac{T_0 L_\Lambda}{T_{\rm reheat} H_{in}^{-1}}\right)\left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{1/3} = \frac{(a_F/a_\Lambda)}{2\pi T_{\rm reheat} H_{in}^{-1}} \cong 10^{25} \tag{9}$$

where we have used the fact that, for a GUTs scale inflation with $E_{GUT} = 10^{14}GeV, T_{\rm reheat} = E_{GUT}, \rho_{in} = E_{GUT}^4$ we have $2\pi H_{in}^{-1} T_{\rm reheat} = (3\pi/2)^{1/2}(E_P/E_{GUT}) \approx 10^5$. If we consider a quantum gravitational, Planck scale, inflation with $2\pi H_{in}^{-1} T_{\rm reheat} = \mathscr{O}(1)$, the phases CE and DF are approximately equal. The region in the quadrilateral CEDF is the most relevant part of standard cosmology, though the evolution of the universe can extend to arbitrarily large stretches in both directions in time.

This figure is definitely telling us something regarding the duality between Planck scale and Hubble scale or between the infrared and ultraviolet limits of the theory. The mystery is compounded by the fact the asymptotic de Sitter phase has an observer dependent horizon and related thermal properties. Recently, it has been shown — in a series of papers, see ref.[39] — that it is possible to obtain classical relativity from purely thermodynamic considerations. It is difficult to imagine that these features are unconnected and accidental; at the same time, it is difficult to prove a definite connection between these ideas and the cosmological constant. I will say more about this in Sections 5, 6.

4. ATTEMPTS ON THE COSMOLOGICAL CONSTANT'S LIFE

4.1. Dark energy from a nonlinear correction term

One of the *least* esoteric ideas regarding the dark energy is that the cosmological constant term in the FRW equations arises because we have not calculated the energy density driving the expansion of the universe correctly. The motivation for such a suggestion arises from the following fact: The energy momentum tensor of the real universe, $T_{ab}(t, \mathbf{x})$ is inhomogeneous and anisotropic and will lead to a very complex metric g_{ab} if only we could solve the exact Einstein's equations $G_{ab}[g] = \kappa T_{ab}$. The metric describing the large scale structure of the universe should be obtained by averaging this exact solution over a large enough scale, to get $\langle g_{ab}\rangle$. But what we actually do is to average the stress tensor *first* to get $\langle T_{ab}\rangle$ and *then* solve Einstein's equations. But since $G_{ab}[g]$ is nonlinear function of the metric, $\langle G_{ab}[g]\rangle \neq G_{ab}[\langle g\rangle]$ and there is a discrepancy. This is most easily seen by writing

$$G_{ab}[\langle g\rangle] = \kappa[\langle T_{ab}\rangle + \kappa^{-1}(G_{ab}[\langle g\rangle] - \langle G_{ab}[g]\rangle)] \equiv \kappa[\langle T_{ab}\rangle + T_{ab}^{corr}] \tag{10}$$

If — based on observations — we take the $\langle g_{ab}\rangle$ to be the standard Friedman metric, this equation shows that it has, as its source, *two* terms: The first is the standard average stress tensor and the second is a purely geometrical correction term $T_{ab}^{corr} = \kappa^{-1}(G_{ab}[\langle g\rangle] - \langle G_{ab}[g]\rangle)$ which arises because of nonlinearities in the Einstein's theory that leads to $\langle G_{ab}[g]\rangle \neq G_{ab}[\langle g\rangle]$. If this term can mimic the cosmological constant at large scales there will be no need for dark energy and — as a bonus — one will solve the coincidence problem! This effect, of course, is real (for an

explicit example, in a different context of electromagnetic plane wave, see [32]) but is usually quite small. In spite of some recent attention this idea has received [33] I doubt whether the idea will lead to the correct result when implemented properly. In a way, the problem is to average physically positive quantities and obtain a result which is not only negative but is sufficiently negative to dominate over positive matter density. One possible way to attack the nonlinear back reaction is to use some analytic approximations to nonlinear perturbations (usually called non-linear scaling relations, see e.g. [14]) to estimate this term. This does not lead to a stress tensor that mimics dark energy (Padmanabhan, unpublished).

4.2. Cosmic Lenz law

The second simplest possibility which has been attempted in the literature several times in different guises is to try and "cancel out" the cosmological constant by some process, usually quantum mechanical in origin. One can, for example, ask whether switching on a cosmological constant will lead to a vacuum polarization with an effective energy momentum tensor that will tend to cancel out the cosmological constant. A less subtle way of doing this is to invoke another scalar field (here we go again!) such that it can couple to cosmological constant and reduce its effective value [34]. Unfortunately, none of this could be made to work properly. By and large, these approaches lead to an energy density which is either $\rho_{\rm UV} \propto L_P^{-4}$ (where L_P is the Planck length) or to $\rho_{\rm IR} \propto L_\Lambda^{-4}$ (where $L_\Lambda = H_\Lambda^{-1}$ is the Hubble radius associated with the cosmological constant). The first one is too large while the second one is too small!

4.3. Unimodular Gravity

One possible way of addressing the issue of cosmological constant is to simply eliminate from the gravitational theory those modes which couple to cosmological constant. If, for example, we have a theory in which the source of gravity is $(\rho + p)$ rather than $(\rho + 3p)$ in Eq. (1), then cosmological constant will not couple to gravity at all. (The non linear coupling of matter with gravity has several subtleties; see eg. [35].) Unfortunately it is not possible to develop a covariant theory of gravity using $(\rho + p)$ as the source. But we can probably gain some insight from the following considerations. Any metric g_{ab} can be expressed in the form $g_{ab} = f^2(x)q_{ab}$ such that $\det q = 1$ so that $\det g = f^4$. From the action functional for gravity

$$A = \frac{1}{16\pi G}\int \sqrt{-g}\, d^4x\,(R - 2\Lambda) = \frac{1}{16\pi G}\int \sqrt{-g}\, d^4x\,R - \frac{\Lambda}{8\pi G}\int d^4x\, f^4(x) \tag{11}$$

it is obvious that the cosmological constant couples *only* to the conformal factor f. So if we consider a theory of gravity in which $f^4 = \sqrt{-g}$ is kept constant and only q_{ab} is varied, then such a model will be oblivious of direct coupling to cosmological constant. If the action (without the Λ term) is varied, keeping $\det g = -1$, say, then one is lead to a *unimodular theory of gravity* that has the equations of motion $R_{ab} - (1/4)g_{ab}R = \kappa(T_{ab} - (1/4)g_{ab}T)$ with zero trace on both sides. Using the Bianchi identity, it is now easy to show that this is equivalent to the usual theory with an *arbitrary* cosmological constant. That is, cosmological constant arises as an undetermined integration constant in this model [36]. (The same result arises in another, completely different approach to gravity which we will discuss in the next section.)

While this is all very interesting, we still need an extra physical principle to fix the value (even the sign) of cosmological constant . One possible way of doing this, suggested by Eq. (11), is to interpret the Λ term in the action as a Lagrange multiplier for the proper volume of the spacetime. Then it is reasonable to choose the cosmological constant such that the total proper volume of the universe is equal to a specified number. While this will lead to a cosmological constant which has the correct order of magnitude, it has an obvious problem because the proper four volume of the universe is infinite unless we make the spatial sections compact and restrict the range of time integration.

5. GRAVITY FROM THE SURFACE DEGREES OF FREEDOM AND THE COSMOLOGICAL CONSTANT

The failure of simple ideas suggests that the problem of cosmological constant will not allow any 'quick fix' solution and is possibly deeply entrenched in quantum gravity. We have got into trouble because of our naive expectations

related to how gravity couples to the vacuum modes. Hence it is necessary to take a few steps back and review all our ideas related to this.

The issue of cosmological constant is intimately related to two nontrivial questions: (i) How is the *microscopic* structure of the vacuum state modified by gravity ? (ii) What kind of *macroscopic* gravitational field is produced by the vacuum ? It is, for example, well known that the vacuum state depends on the class of observers we are considering [37]. In *any* spacetime, there will exist families of observers (congruence of timelike curves) who will have access to only part of the spacetime. The well known examples are observers at $r =$ constant$> 2M$ in the Schwarzschild spacetime or the uniformly accelerated observers in flat spacetime. Any class of observers, of course, has an equal right to describe physical phenomena entirely in terms of the variables defined in the regions accessible to them. The action functional describing gravity, used by these observers (who have access to only part of the spacetime) should depend only on the variables defined on the region accessible to them, including the boundary of this region. (This is essentially the philosophy of renormalization group theory translated from momentum space into real space.) Since the horizon and associated boundaries may exist for some observers but not for others, this brings up a new level of observer dependence in the action functional describing the theory, though the existence of horizons. *The physics of the region blocked by the horizon will be then encoded in a boundary term in the action.*

In fact, it is possible to obtain (see the first paper in Ref. [39]) the dynamics of gravity from an approach which uses *only* the surface term of the Hilbert action; *we do not need the bulk term at all*!. In this approach, the action functional for the continuum spacetime is

$$A_{tot} = A_{sur} + A_{matter} = \frac{1}{16\pi G}\int_{\partial\mathscr{V}} d^3x\sqrt{-g}n_c Q_a^{\ bcd}\Gamma^a_{bd} + \int_{\mathscr{V}} d^4x\sqrt{-g}L_m(g,\phi) \tag{12}$$

where $Q_a^{\ bcd} = (1/2)(-\delta^c_a g^{bd} + \delta^d_a g^{bc})$. Matter degrees of freedom live in the bulk $\mathscr{V}$ while the gravity contributes on the boundary $\partial\mathscr{V}$. When the boundary has a part which acts as a horizon for a class of observers, we demand that the action should be invariant under virtual displacements of this horizon. *This leads to Einstein's theory with a cosmological constant that arises as an integration constant* [39]. We will call this the holographic dual description of standard Einstein gravity. (The term holography is used by different people in different contexts; when I use that term in this talk, I use it in the sense described above.)

What is more important is that, in this approach, gravity has a thermodynamic interpretation. The A_{sur} is directly related to the (observer dependent horizon) entropy and its variation, when the horizon is moved infinitesimally, is equivalent to the change in the entropy dS due to virtual work. The variation of the matter term contributes the PdV and dE terms and the entire variational principle is equivalent to the thermodynamic identity

$$TdS = dE + PdV \tag{13}$$

applied to the changes when a horizon undergoes a virtual displacement. In the case of spherically symmetric spacetimes, for example, it can be *explicitly* demonstrated [40] that the Einstein's equations follow from the thermodynamic identity applied to horizon displacements.

In this approach, the continuum spacetime is like an elastic solid ('Sakharov paradigm'; see e.g. [41]) with Einstein's equations providing the macroscopic description. In the virtual displacement $x^a \to \overline{x}^a = x^a + \xi^a$ the $\xi^a(x)$ is similar to the displacement vector field used, for example, in the study of elastic solids. The true degrees of freedom are some unknown 'atoms of spacetime' but in the continuum limit, the displacement $x^a \to \overline{x}^a = x^a + \xi^a(x)$ captures the relevant dynamics, just like in the study of elastic properties of the continuum solid. Further, it can be shown that the horizons in the spacetime are similar to defects in the solid so that their displacement costs entropy. This suggests that *the true degrees of freedom of gravity for a volume $\mathscr{V}$ reside in its boundary $\partial\mathscr{V}$* — a point of view that is strongly supported by the study of horizon entropy, which shows that the degrees of freedom hidden by a horizon scales as the area and not as the volume. The cosmological constant arises as a undetermined integration constant but closely related to the 'bulk expansion' of the solid.

There is actually a deep reason as to why this works, which actually goes beyond the Einstein's theory. Similar results exists for *any theory based on principle of equivalence*, in which the gravity is described by a metric tensor g_{ab}. Let me briefly describe the general setting from which this thermodynamic picture arises [42].

Consider a (generalized) theory of gravity in D-dimensions based on a generally covariant scalar lagrangian L which is a functional of the metric g^{ab} and curvature $R^a_{\ bcd}$. Instead of treating $[g^{ab}, \partial_c g^{ab}, \partial_d\partial_c g^{ab}]$ as the independent variables, it is convenient to use $[g^{ab}, \Gamma^i_{kl}, R^a_{\ bcd}]$ as the independent variables. The curvature tensor $R^a_{\ bcd}$ can be expressed entirely in terms of Γ^i_{kl} and $\partial_j\Gamma^i_{kl}$ and is *independent* of g^{ab}. It is also useful to define the tensor $P_a^{\ bcd} \equiv$

$(\partial L/\partial R^a{}_{bcd})$ which has exactly the same symmetries of $R^a{}_{bcd}$. Varying the action functional gives

$$\delta A = \delta \int_{\mathscr{V}} d^D x \sqrt{-g} L = \int_{\mathscr{V}} d^D x \sqrt{-g} E_{ab} \delta g^{ab} + \int_{\mathscr{V}} d^D x \sqrt{-g} \nabla_j \delta v^j \tag{14}$$

where

$$E_{ab} \equiv \left(\frac{\partial \sqrt{-g} L}{\partial g^{ab}} - 2\sqrt{-g} \nabla^m \nabla^n P_{amnb} \right) \tag{15}$$

and

$$\delta v^j \equiv [2P^{ibjd}(\nabla_b \delta g_{di}) - 2\delta g_{di}(\nabla_c P^{ijcd})] \tag{16}$$

This result is completely general. In δA in Eq. (14), the second term will lead to a surface contribution. To have a good variational principle leading to the result E^{ab} = matter source terms, we need to assume that $n_a \delta v^a = 0$ on $\partial \mathscr{V}$ where n_a is the normal to the boundary. In general this requires a particular combination of the "coordinates" $[g_{ab}]$ and the "momenta" $[\nabla_c \delta g_{ab}]$ to vanish and we need to put conditions on both the dynamical variables *and* their derivatives on the boundary. It is more reasonable in a quantum theory to choose either the variations of coordinates or those of momenta to vanish rather than a linear combination. To achieve this, let us concentrate on a subset of Lagrangians for which P^{abcd} is divergence free. That is, we demand:

$$\nabla_c P^{ijcd} = 0 \tag{17}$$

Because of the symmetries, this means P^{abcd} is divergence-free in *all* indices. Then Eq. (15) shows that the source-free equations of motion $E_{ab} = 0$ reduces to

$$\frac{\partial \sqrt{-g} L}{\partial g^{ab}} = 0 \tag{18}$$

That is, just setting the *ordinary derivative* of lagrangian density with respect to g^{ab} to zero will give the equations of motion!.

Interestingly enough, this condition encompasses *all* the gravitational theories (in D dimensions) in which the field equations are no higher than second degree, though we did *not* demand that explicitly. To see this, let us consider the possible fourth rank tensors P^{abcd} which (i) have the symmetries of curvature tensor; (b) are divergence-free; (iii) made from g^{ab} and $R^a{}_{bcd}$. If we do not use the curvature tensor, then we have just one choice made from metric:

$$P_a{}^{bcd} = \frac{1}{2}(\delta^c_a g^{bd} - \delta^d_a g^{bc}) \tag{19}$$

This leads to the Einstein-Hilbert action

$$L \equiv P_a{}^{bcd} R^a{}_{bcd} + \text{constant} \equiv R - 2\Lambda \tag{20}$$

with an integration constant which is the cosmological constant. It is easy to verify that the standard Einstein's field equations arise from the *ordinary derivative* taken in Eq. (18) with the variables as we have chosen: $\sqrt{-g}L = \sqrt{-g}[g^{ab}R_{ab} - 2\Lambda]$.

Next, if we allow for $P_a{}^{bcd}$ to depend linearly on curvature, then we have the following additional choice of tensor with required symmetries:

$$P^{abcd} = R^{abcd} - G^{ac}g^{bd} + G^{bc}g^{ad} + R^{ad}g^{bc} - R^{bd}g^{ac} \tag{21}$$

(In four dimensions, this tensor is essentially the double-dual of R_{abcd} and in any dimension can be obtained from R_{abcd} using the alternating tensor [43].) In this case, integrating $(\partial L/\partial R^a{}_{bcd}) = P_a{}^{bcd}$, we get

$$L = \frac{1}{2}\left(g_{ia}g^{bj}g^{ck}g^{dl} - 4g_{ia}g^{bd}g^{ck}g^{jl} + \delta^c_a \delta^k_i g^{bd} g^{jl} \right) R^i{}_{jkl} R^a{}_{bcd} = \frac{1}{2}\left[R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2 \right] \tag{22}$$

(plus, of course, the cosmological constant which we have not exhibited). This just the Gauss-Bonnet(GB) action which is a pure divergence in 4 dimensions but not in higher dimensions. The unified procedure for deriving Einstein-Hilbert action and GB action [essentially from the condition in Eq. (17)] shows that they are more closely related to each other than previously suspected. The fact that *several string theoretical models get GB type terms as corrections* is noteworthy in this regard.

In fact, the holographic dual description of gravity — in which the same equations arise from a surface term — exists for all these theories. To obtain this, note that any nontrivial scalar lagrangian built from $R^a{}_{bcd}$ can be written in the form $L = Q_a{}^{bcd} R^a{}_{bcd}$ without loss of generality. (The tensor $Q_a{}^{bcd}$ depends on curvature and metric; so we have not made any restrictive assumptions [44]). Using the antisymmetry of $Q_a{}^{bcd}$ in c, d we can write:

$$\sqrt{-g}L = \sqrt{-g}Q_a{}^{bcd} R^a{}_{bcd} = 2\sqrt{-g}Q_a{}^{bcd}[\partial_c \Gamma^a_{db} + \Gamma^a_{ck}\Gamma^k_{db}] \tag{23}$$

We now do an integration by parts, use $\partial_c \ln\sqrt{-g} = \Gamma^k_{ck}$ and express $\partial_c Q_a{}^{bcd}$ in terms of $\nabla_c Q_a{}^{bcd}$ and four terms of type ΓQ. This gives a remarkably simple result:

$$\sqrt{-g}L = 2\partial_c\left[\sqrt{-g}Q_a{}^{bcd}\Gamma^a_{bd}\right] + 2\sqrt{-g}Q_a{}^{bcd}\Gamma^a_{dj}\Gamma^j_{bc} - 2\sqrt{-g}\Gamma^a_{bd}\nabla_c Q_a{}^{bcd} \tag{24}$$

So far we have not made any assumptions and this result shows that any scalar gravitational lagrangian built from metric and curvature has a separation into a surface term (first term) and bulk term (second and third terms) in a natural but non covariant manner. Ignoring the surface term, one can obtain the same *covariant* equations of motion as before even from a *non covariant* lagrangian. In general the equations of motion will be higher than second order but let us again specialize to the case in which $Q_a{}^{bcd}$ is divergence-free. Then the last term Eq. (24) vanishes and we get the simple result:

$$\sqrt{-g}L = 2\partial_c\left[\sqrt{-g}Q_a{}^{bcd}\Gamma^a_{bd}\right] + 2\sqrt{-g}Q_a{}^{bcd}\Gamma^a_{dk}\Gamma^k_{bc} \equiv L_{\rm sur} + L_{\rm bulk} \tag{25}$$

which — in particular — should hold for *both* Einstein-Hilbert and GB actions. The second term is the generalisation of the standard Γ^2 action for GB action. In the case of both Einstein-Hilbert action and GB action one can take $Q_a{}^{bcd} = P_a{}^{bcd}$ with suitable normalization. When $Q_a{}^{bcd}$ is built from metric alone, it is given by Eq. (19) and Eq. (25) becomes

$$\sqrt{-g}L = \partial_c\left[\sqrt{-g}(g^{bd}\Gamma^c_{bd} - g^{bc}\Gamma^a_{ba})\right] + \sqrt{-g}(g^{bd}\Gamma^a_{dj}\Gamma^j_{ba} - g^{bc}\Gamma^a_{aj}\Gamma^j_{bc}) \tag{26}$$

which is precisely the bulk-surface decomposition for Einstein-Hilbert action. In the case of GB action, we get a similar result with $Q_a{}^{bcd}$ being given by the right hand side of Eq. (21).

There is also another striking relation between the surface and bulk terms in the lagrangian in Eq. (25). To see this we begin by noting that, the two parts:

$$L_{\rm bulk} = 2\sqrt{-g}\,Q_a{}^{bcd}\Gamma^a_{dk}\Gamma^k_{bc}; \qquad L_{\rm sur} = 2\partial_c\left[\sqrt{-g}Q_a{}^{bcd}\Gamma^a_{bd}\right] \equiv \partial_c[\sqrt{-g}V^c] \tag{27}$$

are both contain the same information in terms of $Q_a{}^{bcd}$ and hence could be *always* related to each other. It is easy to verify [38] that

$$L_{\rm sur} = -\frac{1}{2}\partial_c\left(\delta^k_b \frac{\partial L_{\rm bulk}}{\partial \Gamma^k_{cb}}\right)_{g,R} \tag{28}$$

In the case of Einstein-Hilbert action, there is a still simpler relation:

$$[(D/2)-1]L_{\rm sur} = -\partial_a\left(g_{bc}\frac{\partial L_{\rm bulk}}{\partial(\partial_a g_{bc})}\right) \tag{29}$$

(For the physical significance of this structure, see the papers in [39].)

Finally, probably truer to the term of holography, there is a relation that determines the form of L_{bulk} and L if we know the form of L_{sur} or — equivalently — the V^c. We have:

$$L = \frac{1}{2}R^a{}_{bcd}\left(\frac{\partial V^c}{\partial \Gamma^a_{bd}}\right)_{g,R}; \qquad L_{bulk} = \sqrt{-g}\left(\frac{\partial V^c}{\partial \Gamma^a_{bd}}\right)_{g,R}\Gamma^a_{dk}\Gamma^k_{bc} \tag{30}$$

The first relation also shows that $(\partial V^c/\partial\Gamma^a_{bd})$ is generally covariant in spite of the appearance. This realtion makes the action intrinsically holographic with the surface term containing an equivalent information as the bulk.

There are two comments worth making about the above derivation: First, much of the index gymnastics can be eliminated by introducing a set of tetrads $e^c_{(k)}$ where $k = 0, 1, ..., D$ identifies the vector and c indicates the component. The dual basis is given by $e_d^{(k)}$ with $e_d^{(k)} e^c_{(k)} = \delta^c_d$. Writing $R^c{}_{dba} = e_d^{(k)} \nabla_{[b} \nabla_{a]} e^c_{(k)}$ our Lagrangian becomes

$$L = Q_c{}^{dba} R^c{}_{dba} = 2Q_c{}^{dba} e_d^{(k)} \nabla_b \nabla_a e^c_{(k)} = \nabla_b \left(2Q_c{}^{dba} e_d^{(k)} \nabla_a e^c_{(k)}\right) - 2Q_c{}^{dba} \left(\nabla_b e_d^{(k)}\right) \left(\nabla_a e^c_{(k)}\right) \tag{31}$$

where we have done an integration by parts and used $\nabla_b Q_c{}^{dba} = 0$. This reduces to Eq. (25) in a coordinate basis with $e^c_{(k)} = \delta^c_k, \nabla_a e^c_{(k)} = \Gamma^c_{ak}, \nabla_b e_d^{(k)} = -\Gamma^k_{bd}$. One can also express Eq. (28) in a similar manner.

Second, note that Eq. (25) with $\nabla_b Q_a{}^{bcd} = 0$ represents the most general effective lagrangian for gravity which is consistent with principle of equivalence, general covariance and the dynamical requirement that a well-defined variational principle should exist. The structure of the theory is specified by a single divergence-free fourth rank tensor $Q_a{}^{bcd}$ having the symmetries of the curvature tensor. The semi classical, low energy, action for gravity can now be determined from the derivative expansion of $Q_a{}^{bcd}$ in powers of number of derivatives:

$$Q_a{}^{bcd}(g,R) = \overset{(0)}{Q}_a{}^{bcd}(g) + \alpha \overset{(1)}{Q}_a{}^{bcd}(g,R) + \beta \overset{(2)}{Q}_a{}^{bcd}(g,R^2,\nabla R) + \cdots \tag{32}$$

where $\alpha, \beta, \cdots$ are coupling constants. The first term gives Einstein-Hilbert action and the second one is Gauss-Bonnet action. It is worth recalling *that such a Gauss-Bonnet term arises as the correction in string theories* [45], as to be expected from our general principle. It is also remarkable that any such Lagrangian $L = Q_a{}^{bcd} R^a{}_{bcd}$ with $\nabla_b Q_a{}^{bcd} = 0$ can be decomposed into a surface and bulk terms which are related holographically. (The lagrangian in gauge theories also has similar structure and one can repeat most of the above analysis [46]. This leads to an interesting relationship between gauge theories and gravity though — as is well known — the lack of a metric leads to significant structural dfifferences.)

Everything else goes through as before [in this case, when $\nabla_a Q^{abcd} = 0$] and it is possible to reformulate the theory retaining *only* the surface term for the gravity sector as in the case of Einstein gravity. [For a related but alternative approach, see [47]]. If one considers the infinitesimal virtual displacement $x^a \to x^a + \xi^a$ of the horizon, and use the fact that any scalar density changes by $\delta(\sqrt{-g}S) = -\sqrt{-g}\nabla_a(S\xi^a)$ one can two key results: First is the identity $\nabla_a E^{ab} = 0$ which is just the generalization of Bianchi identity. Second, if we consider an action principle with based on $(A_m + A_s)$ where A_m is the matter action and A_s is the action obtained from $-L_{sur}$ (the minus sign is just to ensure that this is the term which, when *added* to our action will *cancel* the surface term) then, for variations that arise from displacement of a horizon normal to itself, one gets the equation $(E_{ab} - \frac{1}{2}T_{ab})\xi^b\xi^a = 0$ where ξ^a is *null*. Combined with identity $\nabla_a E^{ab} = 0$ this will lead to standard field equations with a cosmological term $E_{ab} = (1/2)T_{ab} + \Lambda g_{ab}$ just as in the case of Einstein-Hilbert action (derived by this route in the first two papers in [39]).

Once again *the cosmological constant arises as an integration constant*. There are three key morals to this story:

- First, the bulk degrees of freedom are gauge redundant and what is really important are the surface degrees of freedom. The bulk cosmological constant which is purely an integration constant should not play any observable role.
- Second, If an observer has a horizon, we should take that seriously, and work with the degrees of freedom confined by the horizon. This clearly changes the pattern of vacuum *fluctuations*.
- Third, and most important, these features arise purely from principle of equivalence and general covariance and is not specific to Einstein's theory. Any theory that has a metric description will have similar features and hence *higher order quantum gravitational corrections are likely to obey these principles.*

I will now show how these ideas lead to a workable model for cosmological constant.

6. A MODEL THAT WORKS: GRAVITY AS DETECTOR OF THE VACUUM FLUCTUATIONS

Finally, I will describe an idea which *does* lead to the correct value of cosmological constant which is based on the following three key ingredients:

- The description of gravity based on purely a surface term in the action provides a natural back drop for ignoring the bulk value of the cosmological constant. This is consistent with the fact that in this approach, bulk

cosmological constant arises as an integration constant.

- What is observable through gravitational effects, in the correct theory of quantum gravity, should be the *fluctuations* in the vacuum energy and *not* the absolute value of the vacuum energy.
- These fluctuations will be non-zero if the universe has a deSitter horizon which provides a confining volume.

Let me now elaborate on this idea. The conventional discussion of the relation between cosmological constant and vacuum energy density is based on evaluating the zero point energy of quantum fields with an ultraviolet cutoff and using the result as a source of gravity. Any reasonable cutoff will lead to a vacuum energy density $\rho_{\rm vac}$ which is unacceptably high. This argument, however, is too simplistic since the zero point energy — obtained by summing over the $(1/2)\hbar\omega_k$ — has no observable consequence in any other phenomena and can be subtracted out by redefining the Hamiltonian. The observed non trivial features of the vacuum state of QED, for example, arise from the *fluctuations* (or modifications) of this vacuum energy rather than the vacuum energy itself. This was, in fact, known fairly early in the history of cosmological constant problem and, in fact, is stressed by Zeldovich [48] who explicitly calculated one possible contribution to *fluctuations* after subtracting away the mean value. This suggests that we should consider the fluctuations in the vacuum energy density in addressing the cosmological constant problem.

If the vacuum probed by the gravity can readjust to take away the bulk energy density $\rho_{\rm UV} \simeq L_P^{-4}$, quantum *fluctuations* can generate the observed value $\rho_{\rm DE}$. One of the simplest models [49] which achieves this uses the fact that, in the semiclassical limit, the wave function describing the universe of proper four-volume $\mathscr{V}$ will vary as $\Psi \propto \exp(-iA_0) \propto \exp[-i(\Lambda_{\rm eff}\mathscr{V}/L_P^2)]$. If we treat $(\Lambda/L_P^2, \mathscr{V})$ as conjugate variables then uncertainty principle suggests $\Delta\Lambda \approx L_P^2/\Delta\mathscr{V}$. If the four volume is built out of Planck scale substructures, giving $\mathscr{V} = NL_P^4$, then the Poisson fluctuations will lead to $\Delta\mathscr{V} \approx \sqrt{\mathscr{V}}L_P^2$ giving $\Delta\Lambda = L_P^2/\Delta\mathscr{V} \approx 1/\sqrt{\mathscr{V}} \approx H_0^2$. (This idea can be a more quantitative; see [49]).

Similar viewpoint arises, more formally, when we study the question of *detecting* the energy density using gravitational field as a probe. Recall that an Unruh-DeWitt detector with a local coupling $L_I = M(\tau)\phi[x(\tau)]$ to the *field* ϕ actually responds to $\langle 0|\phi(x)\phi(y)|0\rangle$ rather than to the field itself [37]. Similarly, one can use the gravitational field as a natural "detector" of energy momentum tensor T_{ab} with the standard coupling $L = \kappa h_{ab}T^{ab}$. Such a model was analysed in detail in ref. [50] and it was shown that the gravitational field responds to the two point function $\langle 0|T_{ab}(x)T_{cd}(y)|0\rangle$. In fact, it is essentially this fluctuations in the energy density which is computed in the inflationary models [7] as the seed *source* for gravitational field, as stressed in ref. [9]. All these suggest treating the energy fluctuations as the physical quantity "detected" by gravity, when one incorporates quantum effects. If the cosmological constant arises due to the energy density of the vacuum, then one needs to understand the structure of the quantum vacuum at cosmological scales. Quantum theory, especially the paradigm of renormalization group has taught us that the energy density — and even the concept of the vacuum state — depends on the scale at which it is probed. The vacuum state which we use to study the lattice vibrations in a solid, say, is not the same as vacuum state of the QED.

In fact, it seems *inevitable* that in a universe with two length scale L_Λ, L_P, the vacuum fluctuations will contribute an energy density of the correct order of magnitude $\rho_{\rm DE} = \sqrt{\rho_{\rm IR}\rho_{\rm UV}}$. The hierarchy of energy scales in such a universe, as detected by the gravitational field has [30, 51] the pattern

$$\rho_{\rm vac} = \frac{1}{L_P^4} + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^2 + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^4 + \cdots \tag{33}$$

The first term is the bulk energy density which needs to be renormalized away by an ad hoc process in the *conventional* approaches. But in the approach outlined in the last section, we can ignore this because gravity is described by purely surface term in action. The third term is just the thermal energy density of the deSitter vacuum state; what is interesting is that quantum fluctuations in the matter fields *inevitably generate the second term*.

The key new ingredient arises from the fact that the properties of the vacuum state depends on the scale at which it is probed and it is not appropriate to ask questions without specifying this scale. If the spacetime has a cosmological horizon which blocks information, the natural scale is provided by the size of the horizon, L_Λ, and we should use observables defined within the accessible region. The operator $H(<L_\Lambda)$, corresponding to the total energy inside a region bounded by a cosmological horizon, will exhibit fluctuations ΔE since vacuum state is not an eigenstate of *this* operator. The corresponding fluctuations in the energy density, $\Delta\rho \propto (\Delta E)/L_\Lambda^3 = f(L_P, L_\Lambda)$ will now depend on both the ultraviolet cutoff L_P as well as L_Λ. To obtain $\Delta\rho_{\rm vac} \propto \Delta E/L_\Lambda^3$ which scales as $(L_P L_\Lambda)^{-2}$ we need to have $(\Delta E)^2 \propto L_P^{-4}L_\Lambda^2$; that is, the square of the energy fluctuations should scale as the surface area of the bounding surface which is provided by the cosmic horizon. Remarkably enough, a rigorous calculation [51] of the dispersion in the

energy shows that for $L_\Lambda \gg L_P$, the final result indeed has the scaling

$$(\Delta E)^2 = c_1 \frac{L_\Lambda^2}{L_P^4} \tag{34}$$

where the constant c_1 depends on the manner in which ultra violet cutoff is imposed. Similar calculations have been done (with a completely different motivation, in the context of entanglement entropy) by several people and it is known that the area scaling found in Eq. (34), proportional to L_Λ^2, is a generic feature [52]. For a simple exponential UV-cutoff, $c_1 = (1/30\pi^2)$ but cannot be computed reliably without knowing the full theory. We thus find that the fluctuations in the energy density of the vacuum in a sphere of radius L_Λ is given by

$$\Delta\rho_{\rm vac} = \frac{\Delta E}{L_\Lambda^3} \propto L_P^{-2} L_\Lambda^{-2} \propto \frac{H_\Lambda^2}{G} \tag{35}$$

The numerical coefficient will depend on c_1 as well as the precise nature of infrared cutoff radius (like whether it is L_Λ or $L_\Lambda/2\pi$ etc.). It would be pretentious to cook up the factors to obtain the observed value for dark energy density. But it is a fact of life that a fluctuation of magnitude $\Delta\rho_{vac} \simeq H_\Lambda^2/G$ will exist in the energy density inside a sphere of radius H_Λ^{-1} if Planck length is the UV cut off. *One cannot get away from it.* On the other hand, observations suggest that there is a ρ_{vac} of similar magnitude in the universe. It seems natural to identify the two, after subtracting out the mean value by hand. Our approach explains why there is a *surviving* cosmological constant which satisfies $\rho_{\rm DE} = \sqrt{\rho_{\rm IR}\rho_{\rm UV}}$ which — in our opinion — is *the* problem.

7. CONCLUSIONS

It is obvious that the existence of a component with negative pressure constitutes a major challenge in theoretical physics. The simplest choice for this component is the cosmological constant; other models based on scalar fields [as well as those based on branes etc. which I did not have time to discuss] do not alleviate the difficulties faced by cosmological constant and — in fact — makes them worse. The key point I want to stress is that the cosmological constant is most likely to be a low energy relic of a quantum gravitational effect or principle and its explanation will require a radical shift in our current paradigm.

I have tried to advertise a new approach to gravity as a possible broad paradigm in which the observed value of the cosmological constant emerges naturally. The conceptual basis for this claim rests on the following ingredients.

- The current problem of cosmological constant is strongly dependent on how we view the gravitational degrees of freedom and its coupling to the vacuum energy. In the new description of gravity based only on the surface term, there are no bulk modes which couples to the vacuum and the *cosmological constant arises as an integration constant*. In such an approach, the bulk value of cosmological constant (the first term in Eq. (33)) is irrelevant.
- The procedure works for a large class of theories (including Gauss-Bonnet type actions in higher dimensions) which are based on principle of equivalence and general covariance. This suggests that the mechanism for ignoring the bulk cosmological constant is likely to survive quantum gravitational corrections which are likely to bring in additional, higher derivative, terms to the action.
- Any generic null surface in a spacetime acts as a horizon for some class of observers. Horizons modify the pattern of vacuum fluctuations and macroscopic gravity acts as a detector of these fluctuations. When computed in a universe with asymptotically deSitter horizon, the vacuum fluctuations inside the horizon lead to the observed value of the cosmological constant (the second term in Eq. (33)).

Getting the correct value of the cosmological constant (the second term in Eq. (33)) is not as difficult as understanding why the bulk value (the first term in Eq. (33) which is larger by 10^{120}!) can be ignored. It is possible to come up with different ad-hoc procedures to do this. I want to emphasize that the holographic approach to gravity provides a natural backdrop for ignoring the bulk term — and as a bonus — we get the right value for the cosmological constant. It is small because it is a purely quantum effect.

This paradigm treats continuum spacetime as analogous to a solid and Einstein's equations as analogous to the elastic dynamics of the solid. This thermodynamic approach acquires surprising support from the results I described in Section 5. The fact that any theory based on principle of equivalence and general covariance can be described by an action principle involving only the surface degrees of freedom cuts right into the heart of the matter. In all these

theories of gravity, cosmological constant will emerge as an integration constant. Considering the generality of this approach, the physical picture should be based on the ability of quantum micro-structure of spacetime to readjust itself absorbing bulk vacuum energy density (like a sponge absorbing water). What one could observe at macroscopic scales is the residual fluctuations (which is like the wetness of the sponge).

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42. Several people asked me about the possibility of this generalisation after my talk at the Paris conference. I thank these participants for pursuading me to study this.
43. One of the early explorations of the properties of this tensor seems to be in E.B. Gliner, GR-5 Meeting, Tbilisi (Proceedings published by the Publishing House of Tbilisi University, Tbilisi (USSR) pp. 86-88.) I thank N.K.Dadhich for bringing this reference to my attention.
44. More directly, note that any function L can be written as $L = (L/2R)(\delta^c_a g^{bd} - \delta^d_a g^{bc})R^a{}_{bcd} = Q_a{}^{bcd} R^a{}_{bcd}$ with $Q_a{}^{bcd} = (L/2R)(\delta^c_a g^{bd} - \delta^d_a g^{bc})$.
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